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What if there are superluminal signals?

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Abstract. Recent experiments with microwaves indicate the existence of signals travelling faster than the vacuum velocity of light. At first glance these signals lead to paradoxical situations which seem to be in conflict with special relativity. We show that the arguments which lead to this contradiction presuppose that superluminal signals are not generally available and that the space-time geometry must be established by means of light rays. Hence we argue that superluminal signals will not necessarily invalidate special relativity.

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1 Introduction

In a recent note by Nimtz [1] and in the literature quoted there it is claimed that microwaves in an undersized wave guide and in photonic lattices can be used for the transmission of superluminal signals. According to these papers the velocity of microwave signals is 5-10 times the velocity of light in vacuum. However, since from a theoretical point of view the velocity in the photonic tunnel barrier is presumably infinite, the measured velocities are average values of the wave propagation inside and at the boundary of the tunnel. The usual reaction to the experiments mentioned is, that these results are not in accordance with special relativity. In the present note we will briefly discuss what actually happens if there are superluminal signals, *i.e.* we want to show how the expected contradiction with special relativity looks like. Finally we discuss the physical relevance of the derived paradoxes.

2 Special relativity

The basic results of special relativity are very well known. Since there is no absolute (universal) time in the sense of Newton, two arbitrary inertial systems I and I′ with Lorentz coordinates (x, t) and (x', t'), respectively, are connected by inhomogeneous Lorentz-transformations [2]. The relative velocity $v = v(\mathrm{I}, \mathrm{I}')$ of these systems turns out to be limited by a universal bound which is numerically identical with the velocity of light in vacuum. Accordingly, the relative velocity of two material observers O and O′ who are connected with systems I and I′, cannot exceed the velocity of light. The same result is obtained by the observation that the inertial mass of a material body depends on its velocity and becomes infinite if the velocity approaches the velocity of light. It should be emphasised that first of all these limitations refer to material bodies and reference systems but not to signals, messages and any kind of information.

3 Superluminal signals

If there were superluminal signals, then these signals could be used for establishing a new space-time metric M′ which is different from the well known Minkowskian metric M which can be realised by light rays and radar-signals [3]. In particular, if there were even instantaneous signals propagating with infinite velocity, Newton's universal and absolute time could be re-established [4]. It is well known, that the unobservability of Newtons absolute time, first mentioned by Ernst Mach [5], was the essential reason for Einstein to formulate special relativity. In fact, special relativity is the theory of space-time which is obtained if one completely dispenses with Newtons absolute time. – If, however, instantaneous signals were possible, by tunnelling or any other means, in the Minkowskian space-time Newtons absolute time could be reconstructed, see [4]. – In order to further illuminate the consequences of superluminal signals we will discuss here two experimental situations which lead to unexpected and at first glance paradoxical results.

i) In an inertial system $\mathrm{I}(x, t)$ with coordinates (x, t) we consider two causally connected events E_1 and E_2 such that $\mathrm{E}_2 \in J^{(+)}(\mathrm{E}_1)$, *i.e.* E_2 lies in the forward light cone $J^{(+)}(\mathrm{E}_1)$ of E_1. Clearly, the chronological order of E_1 and E_2 is invariant against Lorentz transformations and hence identical for all inertial observers. If, however, a superluminal signal S is emitted in E_1 and received in E_2, then the events E_1 and E_2 have space-like distance and the chronological order $t(\mathrm{E}_1) < t(\mathrm{E}_2)$ in system I can be changed if

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the events are described by a moving inertial system $\mathrm{I}'(v)$ with sufficiently large relative velocity v. In this case the signal S is first received and then emitted, *i.e.* we have $t'(\mathrm{E}_1) > t'(\mathrm{E}_2)$. This is in particular the case for instantaneous signals which connect (in $\mathrm{I}(x,t)$) two simultaneous events E_1 and E_2. This paradox can be resolved if the superluminal signals with velocity $c' > c$ are not only used for the transmission of a particular signal from $x = 0$ to $x = L$, but also for establishing a new space-time metric $\mathrm{M}' = \mathrm{M}(c')$ [3]. In the new metric M' the chronological order of the events E_1 and E_2 of sending and receiving the c'-signal will be invariant with respect to Lorentz-transformations written with the fastest velocity c'. This simple argument shows that the paradoxical features of superluminal signals come from the incompatibility of a $(c' > c)$-signal with a space-time theory which is based on a limiting velocity c.

ii) The second and most important argument against the possibility of superluminal signals is that these signals would allow an observer O to send messages into the own past. The Gedankenexperiment, which was first mentioned by Kacser [6] and further discussed in the literature [7] makes use of two inertial observers O and O′ with relative velocity v. Let $\mathrm{I}(x,t)$ and $\mathrm{I}'(x',t')$ be the inertial systems of O and O′, respectively. If observer O sends a signal S from the event point $\mathrm{A} = \{0,0\}_\mathrm{I}$ in I with signal velocity $v_\mathrm{S} = Nc$, $N > 1$, then the signal will be received in $x = L$ in the event $\mathrm{B} = \{L, L/Nc\}_\mathrm{I}$. The system $\mathrm{I}'(x',t')$ of O′ is defined by its velocity v and by the coincidence $\mathrm{B} = \{L, L/Nc\}_\mathrm{I} = \{0,0\}_{\mathrm{I}'}$ of the origin $\{0,0\}_{\mathrm{I}'}$ of I′ with B. In the same instance when O′ receives the signal S, O′ sends a second signal S′ with signal velocity $v_{\mathrm{S}'} = N'c$ back to the observer O who is at rest in $x = 0$. Let $t = t_\mathrm{a}$ be the arrival time of this signal S′ at $x = 0$ then one finds after some elementary calculations

$$t_\mathrm{a} = -\frac{L}{c}\frac{(c - c/N - c/N' + v/NN')}{(c - v/N)}\,. \tag{1}$$

Since for the velocity v of I′ we have the condition $v < c$ the denominator in (1) is always positive. For sufficiently large values of N and N' the numerator will also be positive and thus t_a is negative. Hence, the second signal S′ arrives earlier in $x = 0$ than the first signal was sent off. This is meant by sending signals into ones own past, see Figure 1.

It is interesting to consider some extreme cases of formula (1). Firstly if we put $v = c$ we find for $N > 1$

$$t_\mathrm{a}^{(c)} = -\,(1 - 1/N) < 0\,, \tag{2}$$

i.e. the signal S′ arrives earlier at $x = 0$ than the first signal was sent off. Secondly, we consider the more realistic case $v < c$. If we assume in this case instantaneous signals S and S′, *i.e.* if we put $N = N' = \infty$, then we obtain

$$t_\mathrm{a}^{(\mathrm{inst})} = -Lv/c^2\,. \tag{3}$$

Thirdly, taking both cases together, $v = c$ and $N = N' = \infty$, we obtain the maximal gain of time

$$t_\mathrm{a}^{(\max)} = -L/c\,, \tag{4}$$

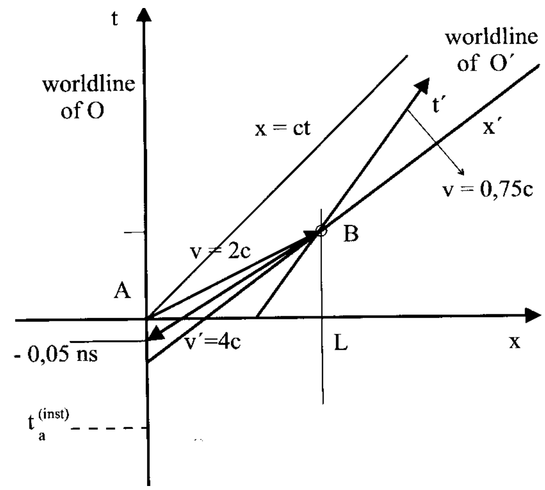

Fig. 1. Two observers O and O′ with coordinates (x,t) and (x',t') respectively. O′ is moving relative to O with velocity $v = 0.75\,c$. O sends a signal S with velocity $v_S = 2c$ from $\mathrm{A} = (0,0)$ to $x = L = 10$ cm. This signal is received in the event $\mathrm{B} = (L, L/2c)$ which lies on the world line of O′. O′ is sending a second signal S′ with velocity $v_S' = 4\,c$ back to O. This signal arrives in $x = 0$ at the arrival time $t_\mathrm{a} = -0.5 \times 10^{-10}$ s, *i.e.* 0.05 ns before O had sent the first signal S.

which can be achieved by instantaneous signalling. In the microwave experiments mentioned [1], we have $L = 10$ cm and thus the maximal gain of time is $t_\mathrm{a}^{(\max)} = 3.33 \times 10^{-10}$ s, a value which is very small for all practical purposes. Irrespective of this almost negligible quantity, the Gedankenexperiment mentioned and the whole way of reasoning must be criticised from a more fundamental point of view. If instantaneous signals are available at all, as it was presupposed in formulas (3, 4), then these signals can also be used for re-establishing Newton's universal time and thus the full Newtonian space-time metric [4]. Inertial systems are then connected by Galileo-transformations. In this prerelativistic space-time the Gedankenexperiment in question looses all paradoxical features. Signal S′ arrives at $x = 0$ exactly at the same time $t = 0$ at which signal S was sent.

4 Conclusion

Summarizing this discussion we find that at first glance superluminal signals provide very strange and paradoxical results. The chronological order of causally connected events can be changed simply by changing the frame of reference. Furthermore, by means of two inertial observers messages can be sent backwards in time. However, the origin of these paradoxical features is that the space-time metric is established by light rays or radar signals and that the superluminal signals are discussed on this Minkowskian background. If space-time were formulated from the very beginning by means of the new superluminal signals, all the paradoxes would immediately disappear. Hence, superluminal signals are not *sui generis* in conflict with special relativity.

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